SIMILARITY IN THE FLOW OF A MAGNETIZED PLASMA AROUND A PLATE AND A CYLINDER

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The article is devoted to a verification of the law of similarity in the flow of a rarefied magnetized plasma around a body under conditions which simulate the conditions of flow around artificial earth satellites in the ionosphere. The law of similarity for flow around plates and cylinders of different sizes ($R_0/\rho_i \approx 0.5-1$, $V_0/V_i \approx 1.5-2$) is confirmed experimentally. It is shown that the patterns of flow around a plate and a cylinder coincide at small values of the parameter $\delta (\delta = R_0/z_H)$. The effect of the potential of the bodies on their flow patterns is studied.

The existence of a similarity law for the flow of a magnetized plasma around bodies of identical profile but of different sizes has been demonstrated theoretically [1]. The examination was conducted in the so-called "neutral approximation" without taking into account the electric field generated by the disturbance of the plasma by the body, so that the results obtained are valid only in the distant zone of the wake. The similarity law is manifested in the fact that the wake profile is described by a function of dimensionless parameters. For example, in flow around a body with a circular cross section of radius R_0 in a uniform magnetic field, the profile of relative densities has the form [1]

$$\frac{n(\rho, z)}{n_0} = f\left(\frac{R_0}{\rho_i}, \frac{\rho}{R_0}, \frac{z}{z_H}\right). \tag{0.1}$$

Here ρ and z are the current coordinates of points in the wake behind the body, n_0 is the density of the undisturbed flow, V_0 is the velocity of flow of the plasma (in (0.1) it is assumed that the plasma moves along the magnetic field H), $\rho_i = V_i/\omega_i$ is the Larmor radius of the ions, V_i is their thermal velocity, ω_i is the cyclotron frequency of the ions, $z_H = 2\pi V_0/\omega_i$ is the distance in which an ion moving with the velocity V_0 along H completes a full turn along the Larmor orbit.

It follows from (0.1) that profiles of the wake are identical in the coordinates ρ/R_0 and z/z_H if the ratio R_0/ρ_i remains constant.

It should be mentioned that for an unmagnetized plasma the existence of a similarity law has been shown theoretically not only in the neutral approximation [1] but also in an approximation taking into account the electric field and also valid in the near zone of the wake [2]. The existence of a similarity law for an unmagnetized plasma was confirmed experimentally [3].

1. Experimental Conditions. The experiments were conducted on an instrument of the Q-machine type. The possibility of using it for the study of the flow of a magnetized plasma around bodies was examined in detail earlier [4].

The plasma is formed by thermal ionization of potassium on a tungsten ionizer 4 cm in diameter heated to temperatures of $T \gtrsim 2000°K$. The plasma is confined by a magnetic field and represents a cylindrical column bounded at one end by the face of the ionizer and at the other by a cold negatively charged electrode ($U = -7$ V, so that $|eU/T| \gg 1$). The plasma density is practically uniform near the axis of the column in the region with a diameter of ~2.5 cm. The experiments were conducted in the electron layer

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TABLE 1.

Parameter	Experiment	Ionosphere
R_0 / r_D	$30 \div 100 \gg 1$	$50 \div 1000 \gg 1$
R_0 / ρ_e	$120 \div 270 \gg 1$	$100 \div 200 \gg 1$
R_0 / ρ_i	$0.45 \div 1.0$	$0.25 \div 1.0$
V_0 / V_e	$(5 \div 8).10^{-3} \ll 1$	$0.01 \ll 1$
V_0 / V_i	$1.2 \div 2.1$	$0.8 \div 6$
R_0 / l	$2 \cdot 10^{-2} \div 2 \cdot 10^{-3} \ll 1$	$5 \cdot 10^{-2} \div 10^{-4} \ll 1$

TABLE 2.

System	Fig. No.,	designations	$2R_0$, mm	H, Oe	V_0, cm/sec	R_0 / ρ_i	V_0 / V_i
1	1	1	3.5	1000	$1.5 \cdot 10^5$	0.5	1.5
		2	5	700			
		3	7	500			
2			3.5	1600	$1.4 \cdot 10^5$	0.7	1.5
			5	1100			
			7	700			
3			3.5	1600	$2.1 \cdot 10^5$	0.7	2.0
			5	1100			
			7	800			
4	2	1	5	1500	$1.5 \cdot 10^5$	1.0	1.6
		2	7	1100			

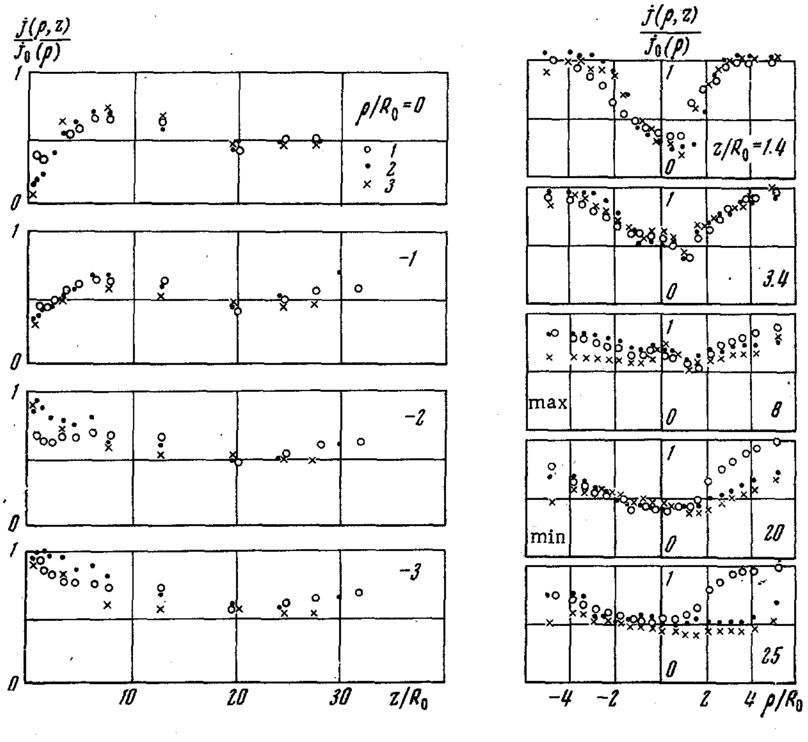


Fig. 1

TABLE 3.

System	Fig. No., designations		$2R_0$, mm	H, Oe	V_0, cm/sec	R_0/ρ_i	V_0/V_i
1	3	*1*	3.5	1600	$2.0 \cdot 10^5$	0.7	2.0
		2	7	800			
2			3.5 7	1400 700	$1.5 \cdot 10^5$	0.6	1.6
3			3.5 7	1000 500	$1.5 \cdot 10^5$	0.5	1.5

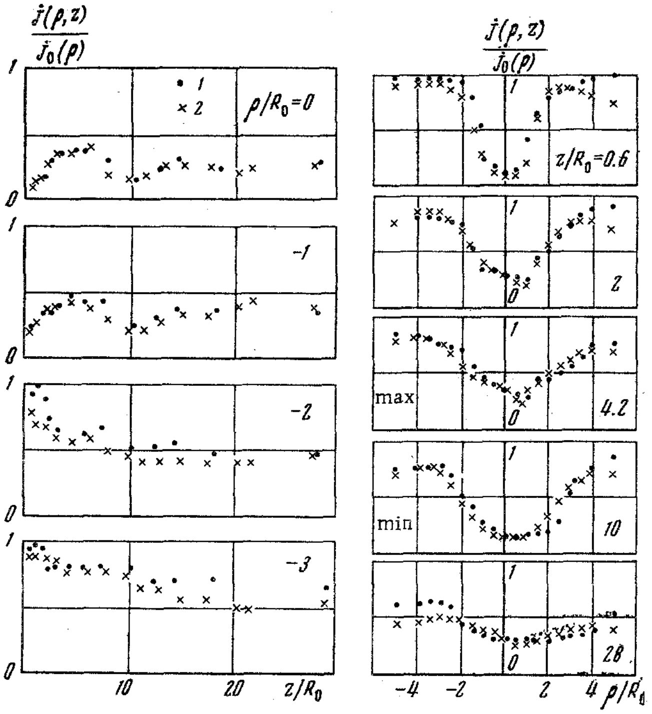


Fig. 2

TABLE 4. Flow around a Plate and a Cylinder

Fig. No.	$2R_0$, mm	H, Oe	V_0, cm/sec	R_0/ρ_i	V_0/V_i	$\delta = R_0/z_H$
4, a	3.5	1600	$1.5 \cdot 10^5$	0.75	1.6	0.076
4, b	3.5	1600	$2.1 \cdot 10^5$	0.70	2.2	0.052
4, c	3.5	1000	$1.5 \cdot 10^5$	0.45	1.5	0.048
4, d	3.5	700	$1.2 \cdot 10^5$	0.35	1.3	0.041
4, e	3.5	700	$1.5 \cdot 10^5$	0.30	1.5	0.033
5	3.5	1600	$1.5 \cdot 10^5$	0.75	1.6	0.076
6	3.5	700	$1.2 \cdot 10^5$	0.35	1.3	0.041
7,8	3.5	1000	$1.3 \cdot 10^5$	0.45	1.3	0.053
—	7	1500	$1.6 \cdot 10^5$	1.4	1.7	0.135
—	7	1100	$1.5 \cdot 10^5$	1.0	1.6	0.11
—	7	700	$1.6 \cdot 10^5$	0.65	1.7	0.063
—	7	800	$2.1 \cdot 10^5$	0.70	2.1	0.062
—	7	500	$1.4 \cdot 10^5$	0.45	1.5	0.048

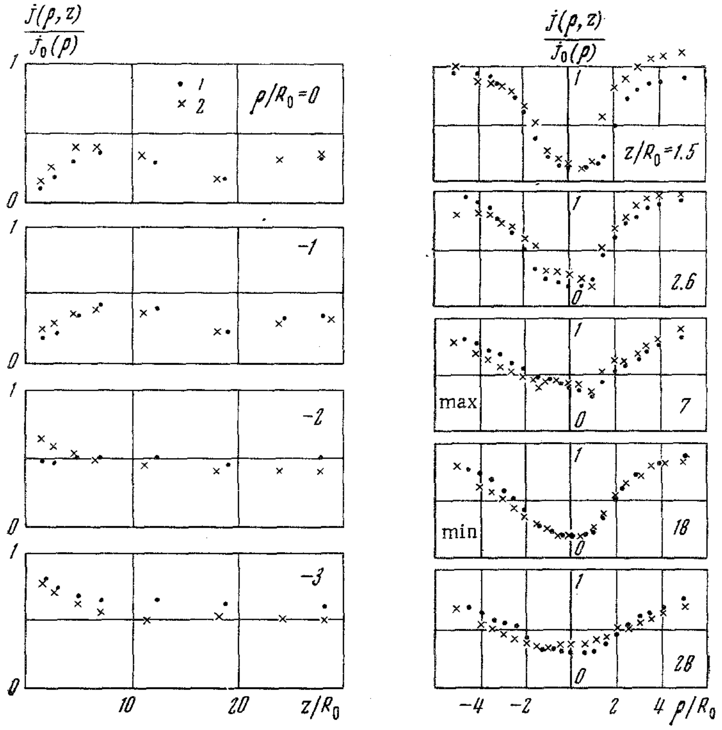


Fig. 3

mode. In this case the ions are accelerated in a layer which acquires a directional velocity V_0 and a longitudinal temperature $T_{\|}$ [5]. The plasma flows from the ionizer to the cold electrode with a velocity V_0. The flow is stable in the electron layer mode.

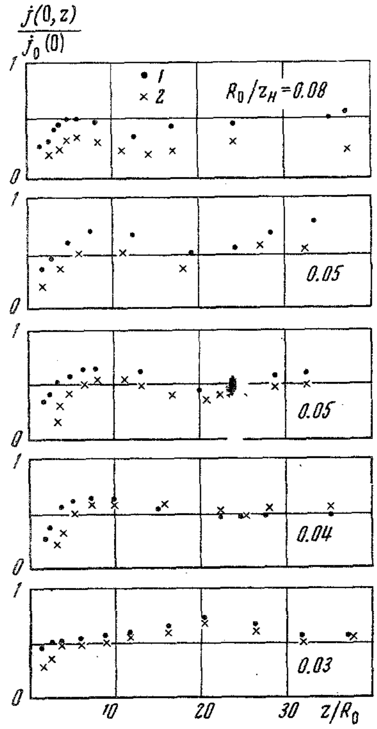


Fig. 4

The electrons have a Maxwell distribution function with a temperature $T_e \sim T$ while the ions have a two-temperature Maxwell function with $T_{\|} < T_{\perp} \sim T$ [4]. The plasma parameters are such that e-i and e-e collisions do not play a role (free path length $l \gtrsim L$), although i-i collisions leading to the equalization of $T_{\|}$ and $T_{\perp}$ can be important.

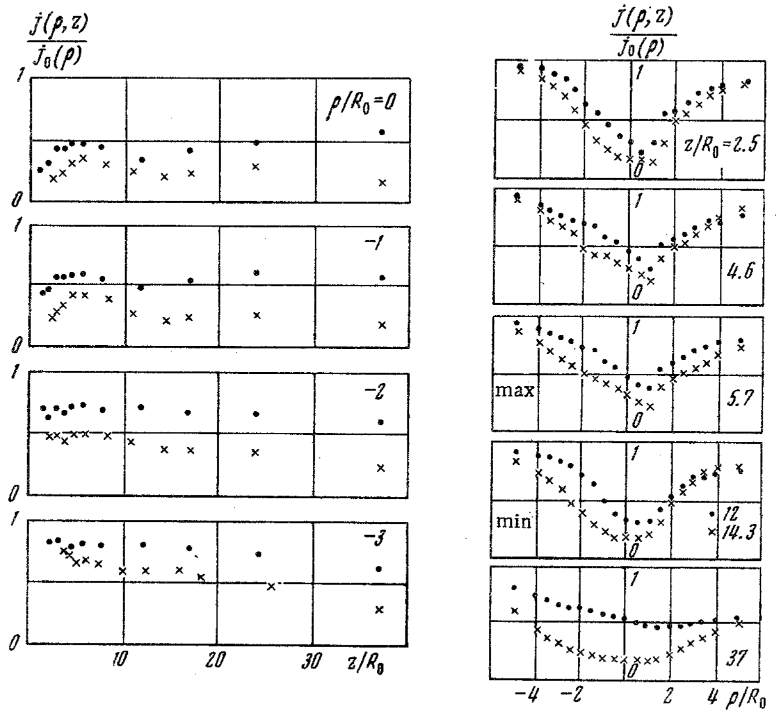
The magnetic field was varied from 500 to 1600 Oe, plasma density $n_0 \sim 1-6 \cdot 10^9$ cm^{-3}, plasma flow velocity $V_0 = (1.2-2.1) \cdot 10^5$ cm/sec, and ionizer temperature T = 2000-2500°K.

The density measurements were conducted with cylindrical tungsten probes 2 mm long and 0.25 mm in diameter; the flow velocity was determined from the period z_H of the longitudinal density oscillations on the axis of the wake behind the flat body [5]

$$V_0 = z_H\omega_i / 2\pi \qquad (1.1)$$

The flow around plates (flat bodies) with a width of $2R_0 = 3.5$, 5 and 7 mm and cylinders (three-dimensional bodies) with a diameter of $2R_0 = 3.5$ and 7 mm was studied. The plates and cylinders were 3 cm high so that a two-dimensional pattern was sufficient to describe the wake. The body studied was placed in the plasma perpendicular to the flow in the zone of uniform density. The experiments were conducted with negatively charged bodies (U = −7 V).

The conditions of these experiments simulate well the flow around bodies ~2 mm in size in the ionosphere at altitudes of ~200-1000 km, which is seen from Table 1 (r_D is the Debye radius and ρ_e and V_e are the Larmor radius and thermal velocity of the electrons).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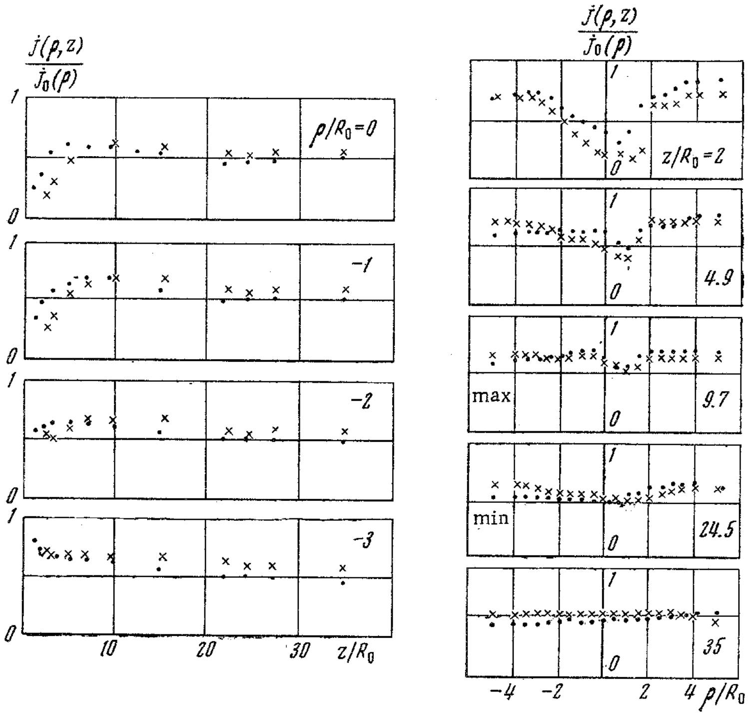
1
0
ρ/R₀=0
−1
−2
−3
10
20
30
z/R₀
z/R₀=2.5
4.6
max
5.7
min
12
14.3
37
−4
−2
2
4
ρ/R₀

Fig. 5


j(ρ,z)/j₀(ρ)
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ρ/R₀=0
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10
20
30
z/R₀
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max
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min
24.5
35
−4
−2
2
4
ρ/R₀

Fig. 6

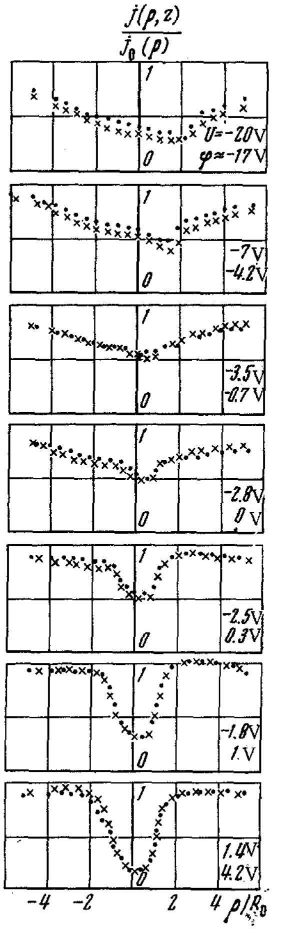

Fig. 7

2. Flow around Bodies of Different Sizes. In order to verify the existence of similarity of flow around identical bodies of different sizes experiments were selected in which constant values of R_0/ρ_i and V_0/V_i are conserved for bodies with different R_0. The parameters of the systems studied are presented in Tables 2 and 3 for plates and cylinders, respectively.

To clarify the question of whether similarity exists, one must construct a cross section of the wake $n(\rho, z)/n_0$ in the coordinates ρ/R_0, z/z_H. Since

$$\frac{z}{z_H}=\frac{1}{2\pi}\frac{z}{R_0}\frac{R_0}{\rho_i}\frac{V_i}{V_0}$$

and $R_0/\rho i$, Vi/V_0 are constant, the coordinate z/z_H can be replaced by z/R_0.

The ratio $n(\rho, z)/n_0$ can be replaced by the ratio of probe points $j(\rho, z)/j_0$ which is equal to it. Distributions of relative density along the wake at different distances from the axis, and distributions of density across the wake at different distances from the body for several experiments, are shown in Figs. 1-3.

In all the experiments the pattern is practically identical for bodies of different sizes in the region within the wake ($\rho/R_0<2$). Significant differences are observed, as a rule, at large ρ/R_0, z/R_0 and do not have a systematic nature. Their source evidently consists in the large measurement errors connected with the insufficient reproducibility of the experimental conditions (the measurements for bodies of different sizes were conducted in different experiments after reassembly of the experimental system).

Thus, it can be confirmed that under the experimental conditions used ($R_0/\rho_i\approx0.5$-1, $V_0/V_i\approx1.5$-2) the similarity law is satisfied both for flat and three-dimensional bodies. Similarity is observed both in the distant zone in accordance with the prediction of the neutral approximation of the theory of [1] and in the near zone, for which a theoretical consideration was not conducted in the case of a magnetized plasma.

3. Flow around a Plate and a Cylinder of Identical Cross Section. A comparison of the patterns of flow around a plate and a cylinder was made. Such a comparison has not been made before for the case of a magnetized plasma. It was shown theoretically for the case of an unmagnetized plasma [2] that the patterns coincide at a large flow velocity of the plasma ($V_0/V_i) \gg 1$).

The results of measurements of the relative density of the plasma on the axis in different systems are presented in Fig. 4. The value

$$\delta=\frac{R_0}{z_H}=\frac{1}{2\pi}\frac{R_0}{\rho_i}\frac{V_i}{V_0} \qquad (3.1)$$

which characterizes the ratio of the longitudinal dimension of the cylinder to the length of the density oscillation in the wake was chosen as a parameter. It is reasonable to expect that the patterns of flow around a plate and a cylinder will be identical when this ratio is small. Cross sections of the wake behind a plate (1) and a cylinder (2) on the axis for different values of the parameter δ are shown in Fig. 4. It is seen that at large values of this parameter the wakes of a plate and a cylinder differ – the density level is lower in the wake of a cylinder. At small values of δ the wakes coincide with the exception of the near zone behind the body.

Total patterns of the wake behind a body (longitudinal and cross sections of the wake) in two systems are presented in Figs. 5 and 6. The parameters of the systems studied are presented in Table 4.

It follows from the experiments that at large values of δ (Fig. 5) the wake of a cylinder lies below the wake of a plate, while at small values of δ (Fig. 6) the wakes practically coincide, with the exception of the near zone. For identical values of δ the wakes obtained with different combinations of R_0, H, and V_0 have an identical nature. Thus, the parameter δ qualitatively characterizes the relative pattern of flow around negatively charged bodies, and the wakes coincide when δ is small, i.e., when the longitudinal dimension of the three-dimensional body is small compared with the length of the oscillation. It should be noted that in the near zone of the wake, the wakes are usually different even at small values of δ (Figs. 4 and 6).

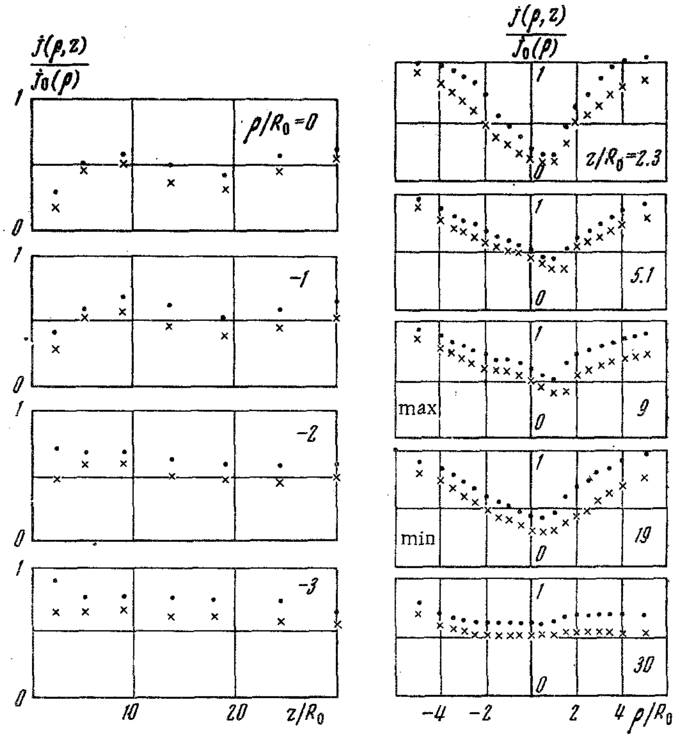

Fig. 8

4. Flow around a Plate and a Cylinder at Different Potentials. The effect of the potential of the body on the flow pattern was studied. In these experiments work with potentials close to the plasma potential presented some difficulty. The point is that the contact potential difference (CPD), produced because of the formation of a film of alkali metal on the body (in work with potassium the CPD can reach 2-3 V) [5], can play an important role in work with alkali plasmas. Since the area of the plate and cylinder is rather large, heterogeneity of the CPD can be important, so that an independent determination of the plasma potential (from the volt–ampere characteristics of the probe, for example) is not very reliable.

It turned out that the shape of the radial cross section of the wake changes considerably with the transition from the region of negative potential to the region of positive potential (see Fig. 7). This is understandable, since with a positive potential on the body, electrons entering the wake are absorbed by the body, so that the cross section of the wake approximates the geometrical cross section of the body. By constructing a set of curves $n(\rho)$ at some cross section for different potentials with a step of 0.1 V, one can determine the transition point with an accuracy of ~0.2 V. This transition potential is also taken as the plasma potential U_0. In the operating system (Fig. 6) it proved to be −2.8 V for the plate and −3.3 V for the cylinder.

The validity of the measured plasma potential can be checked by determining the floating potential U_* and comparing the different U_*-U_0 with the calculated value $\varphi_*-\varphi_0$ of the floating potential relative to the plasma potential [5]. It was found that $U_*-U_0=0.7$ V agrees well with $\varphi_*-\varphi_0=0.8$ V, so that the method adopted makes it possible to determine the plasma potential rather accurately. We note that the absolute value of the plasma potential in the working system is equal to $|\varphi_0| \sim 0.2$ V according to the calculation of [5].

Radial cross sections of the wake at the oscillation maximum for different potentials of the body (U is the potential of the plate, φ is the potential of the body relative to the plasma potential: $\varphi = U-U_0$) are presented in Fig. 7. The change in the nature of the wake upon the transition to the region of positive potentials is clearly seen from the figure. It is seen that the depth and width of the wake increase with an increase in both the positive and negative potential. This shows the important role of the space charge layer at the surface of the charged body. The size of the layer grows with an increase in the potential, i.e., the effective size of the body grows and consequently the width and depth of the wake must increase.

At the plasma potential and more positive potentials and at negative potentials down to the floating potential the wakes of the plate and the cylinder coincide everywhere; at more negative potentials the wake of the cylinder becomes deeper than the wake of the plate (Fig. 8). Here the wake of the the plate at $U=-7$ V hardly differs from the wake at $U=U_0$, while the wake of the cylinder becomes deeper. This indicates the more rapid growth of the effective surface of the layer of the cylinder. With a further increase in the potential the layer begins to have a considerable effect on the wake on the plate (Fig. 7).

Thus, one can distinguish the following regions with an increase in the negative potential:

1) $U=U_0$. With streamline flow the geometrical factors must play the principal role. The agreement or difference of the wakes of a plate and a cylinder must be determined by the parameter δ;

2) $U \lesssim U_0$. The size of the layer is small and the layer has practically no effect on the streamline flow in the distant zone ($U=U_*$ in Fig. 7), but it may show up in the near zone. The parameter δ retains its role;

3) the layer markedly affects the wake of a cylinder but does not affect the wake of a plate ($U=-7$ V in Fig. 7) except for the near zone. One must substitute the effective dimension $R_* > R_0$ for the value R_0 in the parameter δ;

4) the layer markedly affects the wakes of the plate and the cylinder ($U=-20$ V in Fig. 7). In this region the plate also has an effective longitudinal size, so that the comparative pattern becomes more complex and demands special allowance for the effect of the layer.

The limiting potentials for these regions naturally depend on the characteristics of the plasma flow and of the body, such as R_0, ρ_i, V_0, and V_i. For example, for different modes the boundary potentials of regions 3 and 4 are equal to:

δ	0.033 (Fig. 4d)	0.053 (Fig. 7)	0.087
U_3	−10 V	−4 V	~(−3) V
U_4	−15 V	−10 V	~(−7) V.

Thus, for a given potential of the bodies different modes can occur in different regions (for example, at $U=-7$ V the mode with $\delta=0.087$ occurs in region 4, mode 7 in region 3, and mode 4 in region 2). Here the relative pattern of the wakes of a plate and a cylinder must be determined by the parameter $\delta_* = R_*/z_H$, which changes from δ in region 2 to $\delta_* > \delta$ in region 3. The experimental results show that the pattern can be described qualitatively using the parameter δ.

A comparison of the patterns of streamline flow at $U=U_0$ and $U=-7$ V shows that coincidence of the wakes of a plate and a cylinder at $U=-7$ V is observed at lower values of δ than in the case of $U=U_0$. In fact, at $U=U_0$ the wakes differ in the modes with $\delta=0.087$ and 0.073 and coincide in modes 7 and 4d, while at $U=-7$ V the wakes coincide in mode 4d and differ in modes 7 and 5. Thus, at $U=U_0$ the transition parameter lies in the range of $0.07 > \delta > 0.05$, while at $U=-7$ V it lies in the range of $0.05 > \delta > 0.04$. This confirms the assumption that at the plasma potential (region 1) the streamline flow is determined by the parameter δ, while at more negative potentials (region 3) it is determined by the parameter $\delta_* > \delta$.

At $U=U_0$, in contrast to the case of $U=-7$ V, coincidence is observed not only in the far zone but also in the near zone. Thus, it can be assumed that the difference in the wakes of a plate and a cylinder in the near zone at $U=-7$ V in modes where the wakes coincide in the far zone (Fig. 4) is caused by the effect of the space charge layer.

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